TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 562

REMARKS ON THE ELASTIC AXIS OF SHELL WINGS

By Paul Kuhn
Langley Memorial Aeronautical Laboratory

Washington
April 1936

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 562

REMARKS ON THE ELASTIC AXIS OF SHELL WINGS

By Paul Kuhn

SUMMARY

The definitions of flexural center, torsional center, elastic center, and elastic axis are discussed. The calculation of elastic centers is dealt with in principle and a suggestion is made for the design of shear webs.

INTRODUCTION

The use of the elastic axis as a convenient means of aiding in the stress analysis of airplane wings is becoming increasingly general. Although the term "elastic axis" is frequently used, perusal of stress analyses shows that there is little agreement on its exact definition. This lack of agreement entails different methods of computation, different methods of experimental verification, and sometimes differences of opinion concerning the results. In order to clarify the terminology, the terms involved are discussed in the present paper. Some suggestions on design methods are made incidentally.

FLEXURAL CENTER - TORSIONAL CENTER - ELASTIC CENTER

For the sake of simplicity, the case of a cantilever beam of constant section loaded by a single concentrated transverse load or by a couple at some section along the axis will first be considered. The case of a straight cantilever beam with constant section and loaded at the tip is the only one that has as yet received theoretical treatment.

The flexural center of a section is defined as that point of the section through which a concentrated bending load must act in order to produce only bending deflection, no twist at the section (fig. 1).

If a torsional load (a couple) is applied at the same section, the section will rotate about some point of the section. This point is called the "torsional center" (center of shear, center of twist, fig. 2).

For a prismatic bar so loaded that the assumptions of the ordinary theories of bending and torsion hold, the flexural and torsional centers coincide, which can easily be proved by considering the elastic energies of bending and of torsion or by equivalent considerations (reference 1). Actually, these assumptions are always violated at the support as well as at the load. As a result, the two centers do not exactly coincide even in prismatic bars. This fact, obvious from an examination of the assumptions involved, has been experimentally verified. The distance between the two centers depends in practical cases chiefly on the relation between the length of the bar (from support to load) and the dimensions of the cross section.

If the distance between the flexural center and the torsional center is sufficiently small to be neglected for practical purposes, the average location of the two centers will hereinafter be called the "elastic center" of the section. The elastic center is, in practice, calculated as the flexural center, as will be shown later in more detail.

In built-up thin-walled structures, some parts buckle far below the design loads and no longer carry their full share of the load. This condition is taken care of in the design by assuming an "effective section," in which every part carries the full share of the load dictated by the laws of the ordinary theory of bending or of torsion, as the case may be. There is no relation, however, between the effective sections for bending and torsion; both change with the magnitude of the load, and consequently the relation between the flexural center and the torsional center changes with the loading condition. It is evident, then, that in a thin-walled structure subjected to large loads, the assumption of an elastic center may give only a very rough picture of the true conditions.

THE ELASTIC AXIS

The elastic axis of an airplane wing is defined as the spanwise line along which loads must be applied in order to produce only bending, no torsion of the wing at any station along the span (definition 1). We shall now investigate under what conditions an elastic axis may be said to exist. In order to simplify the argument as much as possible, the simple 2-spar wing structure shown in figure 3 will be used as a basis for the discussion. This restriction to a simple type of structure will not invalidate the generality of the conclusion because it will be shown that an elastic axis in the commonly used sense of the word does not exist in the general case.

Let us consider first the load case shown in figure 3, a single concentrated load located at a given distance from the root. Obviously, it will be possible to find a chordwise location for this load such that the deflections of the two spars are equal at the load station, i.e., such that the wing has no twist at the load station. It is equally obvious, however, that there will be a twist at all other stations along the span. Consequently, in the general case of a wing with variable cross section, <u>definition 1 of the elastic axis is inapplicable to the case where only a single concentrated load acts</u>.

Let us consider next the case of a bending load distributed uniformly along the span. A uniformly distributed load acting on the front spar will produce a deflection curve that can be calculated. The condition of no twist in the wing at any station can now be fulfilled by distributing the load on the rear spar in such a manner that the rear spar has the same deflection curve as the front spar. The distribution curve for the load on the rear spar can be calculated from the stiffness properties of the spar; the relation between the load per unit span length on the front and on the rear spars at any station along the span defines the elastic axis of the wing. <u>For any given continuously distributed bending load, an elastic axis can be found</u>.

Now let us replace the uniformly distributed load on the front spar by a different one, say, for example, a load with triangular distribution. Again it will be possible to calculate the deflection curve of the front spar and to calculate a load distribution for the rear spar such that the rear spar has the same deflection curve. The condition of equal deflection curves, however, already completely determines the loading curve on the rear spar;

it is impossible to fulfill at the same time the condition that the relation between front and rear loading curves shall be the same at any station, i.e., the resulting elastic axis for the triangular loading will differ from that for uniform loading. The position of the elastic axis varies if the loading distribution changes.

In the case of the idealized 2-spar wing considered thus far, torsion of the wing is due to differences in the deflection curves of the spars. In a practicable 2-spar wing, the spars would be forced by the ribs to twist, but the torsional stresses arising would be of small importance if the spars had small torsional stiffnesses individually (e.g., I-beams). The importance of the torsional stresses increases as these torsional stiffnesses increase (individual box spars), and the torsional stresses become very important in the case of shell wings. In wings of the shell type a serious difficulty may arise. If the "structural axis" of the wing is curved, it is impossible to separate bending stresses from torsional stresses; the terms "bending" and "torsion" lose their identity. Wings with a curved structural (elastic) axis are not amenable to calculation on the basis of the ordinary theories of bending and torsion. No attempt will therefore be made to define the structural axis in the general case because such a definition would serve no useful purpose.

Finally, attention should also be called to the fact that the ordinary formulas for bending and torsional stresses in any beam do not hold near points of discontinuity in loading (concentrated loads) or in cross section (cut-outs, root-section). The errors due to the use of the ordinary theories, however, become negligible at some distance away from the point of discontinuity if this distance is of the same order of magnitude as the cross-sectional dimensions of the beam.

Definition 1 of the elastic axis has probably never been used to determine the elastic axis of a wing by calculation or by experiment because it involves too much labor. The following definition has been generally substituted: The elastic axis of a wing is the locus of the elastic centers (definition 2).

In view of the conclusion previously reached that definition 1 is not applicable to the case where a single concentrated load acts, the substitution of definition 2 for definition 1 is not justifiable in the general case.

It is strictly correct only in the case of a wing with constant section. In practical cases its use is often permissible, however, if the locus of the centers is at least approximately straight.

To summarize the findings, it may be said that the use of the elastic-axis method of stress analysis can be expected to give reasonably close results only if the following conditions are met:

1. The load distribution must be sensibly the same for the various load cases investigated.

2. The elastic axis must be without large discontinuities and must be sensibly straight (and at right angles to the root section for investigating the region near the root.)

3. Attention must be paid to possible shifting of the elastic axis owing to partial buckling at high stresses.

The foregoing remarks are not intended to discourage entirely the use of the elastic-axis method. They are intended, however, to emphasize that this method is inherently very approximative and must therefore be used with caution and judgment.

CALCULATION OF THE ELASTIC CENTER

As stated before, the problem of calculating the elastic axis of a wing is reduced, for practical purposes, to the problem of calculating the elastic center of any section of the wing.

In a wing with two or more independent spars (fig. 4), a load at any arbitrary point causes mainly bending stresses and bending deflections in the spars. It is obvious that in this case the elastic center is located at the centroid of the bending stiffnesses of the spars, or at the centroid of the moments of inertia if all spars are of the same material and shear deflections are negligible. If the shear deflections are not negligible, corresponding corrections must be made.

This simple method of calculating the elastic center of a spar-type wing has frequently been applied to shell-type wings by dividing the profile into strips (fig. 5) and assuming that the material in each strip acts as an independent spar. This method is fundamentally incorrect. The material at the top and at the bottom in any one strip cannot possibly act as an independent beam unless there is a connecting shear web, a web being an indispensable part of a beam. If there is no shear web in a strip, the shear stress must detour sideways until it does meet a shear web; <u>the total resultant of these shear stresses must coincide with the external shear force acting on the section if there is to be no torsion</u>. The elastic center is therefore determined as the centroid of the shear forces, not of the moments of inertia of the flange material. The detoured components of the shear forces exert a twisting couple, which must be counteracted by offsetting the external load from the point that would have been the elastic center if the proper shear webs had been provided for each strip.

For simple shapes with "statically determinate" distribution of the normal stresses (fig. 6), the elastic center can be quite easily calculated. The load is resolved into a component P in the beam direction $A_1 - A_2$ (shown on the figure) and a component in the chord direction $A_3 - A_2$ (not shown). For the beam component P, the flange A_3 lies on the neutral axis and can be disregarded. The shear due to P is distributed over the nose N and the web W; the portion carried by each is determined by the condition that the shear strain of the nose part N between A_1 and A_2 must be equal to the shear strain of the web W between the same two points (reference 2), or

$$f_{s_N}\ s_N = f_{s_W}\ s_W$$

(assuming nose and web to be of the same material). Two elementary integrations show that the resultant shear force on the nose is (fig. 7)

$$V = f_{s_N}\ h\ t_N$$

and is located at the distance

$$e = \frac{2A}{h}$$

from the plane of the web.

In these equations f_s is the shear stress; s, the perimeter as shown in figure 6; h, the height of the nose part (fig. 7); t, the wall thickness; and A, the area included by the profile. The subscripts N and W refer to the nose and web parts, respectively.

Locating the resultant of the shear forces in nose and web then gives one line on which the elastic center lies; repetition of the process for the chord component gives a second line; the elastic center is thus determined.

For more complicated arrangements of the flange material, the calculation becomes more involved and more doubtful because the distribution of the normal stresses in such built-up sections is not very well established. An idea of the possible errors may be obtained from the fact that a number of careful tests and calculations (reference 3) on comparatively simple shapes similar to figure 6 gave discrepancies in the location of the elastic axis of about 15 percent of the box chord or about 5 percent of the wing chord. These calculations were made more carefully than usual and for some of them the shearing stiffnesses of the webs were first experimentally determined; it therefore seems safe to conclude that routine calculations of elastic centers for wings as shown in figure 5 cannot at present be expected to give the location of the elastic center with a possible error much less than 10 percent of the wing chord unless careful corrections based on tests of very similar wings are made.

The fundamental factor determining the distribution of the shearing stresses and consequently the location of the elastic center is the shear strain of the component parts, which in turn depends on the shearing stiffness of these parts. For a plate without lightening holes and heavy enough to withstand buckling, or for truss-type webs, the shearing stiffness is independent of the load, i.e., the strain is proportional to the load applied until the proportional limit of the material is reached. For webs with round or oval lightening holes and for webs that buckle into diagonal-tension fields, there is no proportionality between strain and load; it would seem very desirable to take this fact into account by using the shearing stiffnesses actually prevailing at the design loads. The best way to obtain the load-deformation curve of a proposed type of web is to build up and test a box as sketched in figure 8. The walls of the box should consist of identical specimens of the web and the length should be sufficient to give an easily measured twist. The effects of restrictions due to the load arms should be reduced by making the distance between the pointers and the load arms sufficiently large (about twice the depth of the box).

It might be remarked here that tests on wings with two or more shear webs have shown that the trailing-edge portion aft of the rear shear web is practically ineffective in bending as well as in torsion and that it should be neglected in the analysis. The nose portion also loses much of its efficiency at high loads but not enough to justify neglecting it.

When the load-deformation diagram is curved, the term "stiffness" needs to be defined. Inasmuch as the state of stress considered is assumed to be reached by increasing the load continuously from a low value to the design load, it seems reasonable to use the average stiffness between zero load and design load and to define it as the constant stiffness that the member should have in order to store the same amount of strain energy as the actual member when loaded to the design load. There are objections to this procedure, but it tends to create a design having uniform stiffness properties in addition to uniform strength throughout the structure. It is conceivable, of course, that such uniformity may not be desirable in certain exceptional cases.

DESIGN OF SHEAR WEBS

One of the most vexing problems of shell-wing design is the question of proportioning the shear webs. Several methods of design and analysis are now in use but none is strictly rational. Pending a more thorough investigation of the problem by theory and experiment, the following suggestions may be made.

Shell wings are now and probably for some time to come will be analyzed under the assumption that they obey the laws of the ordinary theory of bending; i.e., that the normal stress is proportional to the distance from the neutral axis. The distribution of the flange material absorbing the bending stresses is fixed by the designer, who is guided by manufacturing or structural considerations. In order to insure maximum structural efficiency, the shear webs should now be designed so that the section obeys as closely as possible the assumed laws of bending. This end is achieved by making the effective shear stiffness of each web proportional to the bending stiffness of the flange material belonging to the web.

The exact amount of flange material to be assigned to each web is somewhat uncertain. Stringers located some distance away from shear webs are less efficient than those located close to shear webs, owing to the shear deformation of the skin. For the present purpose, however, it is probably sufficiently accurate to draw the division lines halfway between the shear webs, unless the skin thickness changes between webs or the depths of adjacent webs are very different.

The shearing stiffness of the web may be experimentally determined, if necessary, as discussed under Calculation of the Elastic Center. If webs of different characteristics are employed in one wing (e.g., solid webs and lightened webs), it may, of course, be necessary to modify the design obtained from considerations of stiffness because one type of web may show permanent set or local failures.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., February 26, 1936.

REFERENCES

1. Timoshenko, S.: Strength of Materials. Part. II. D. Van Nostrand Co., Inc., 1930.

2. Wagner, Herbert: Remarks on Airplane Struts and Girders under Compressive and Bending Stresses, Index Values. T.M. No. 500, N.A.C.A., 1929.

3. Hertel, Heinrich: Die Verdrehsteifigkeit und Verdrehfestigkeit von Flugzeugbauteilen. D.V.L, Jahrbuch 1931, pp. 165-220.

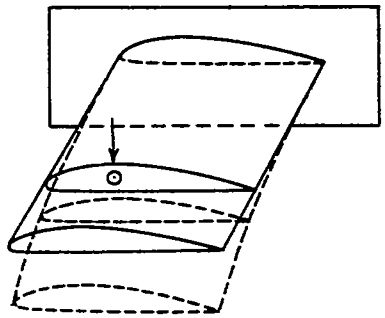

Figure 1.- Flexural center.

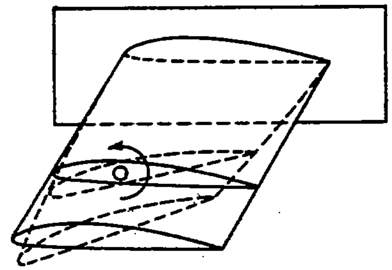

Figure 2.- Torsional center.

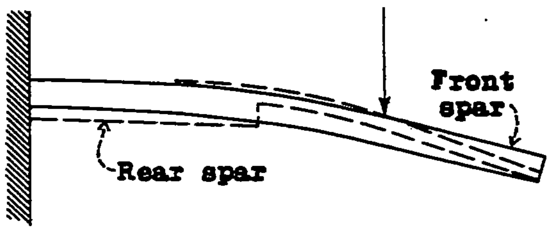

Figure 3.

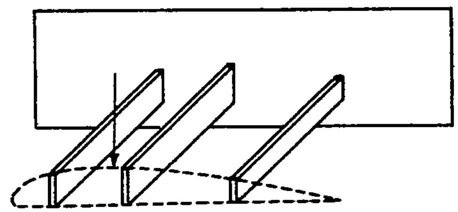

Figure 4.

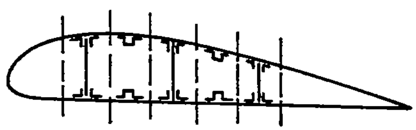

Figure 5.

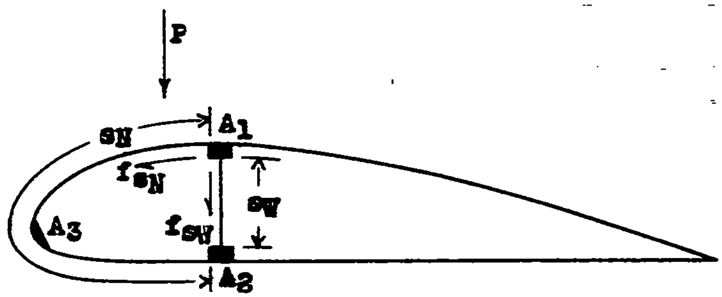

Figure 6.

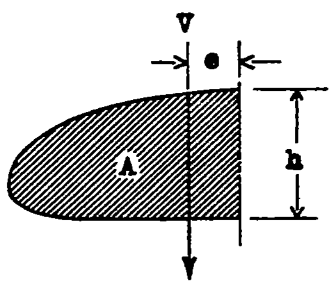

Figure 7.

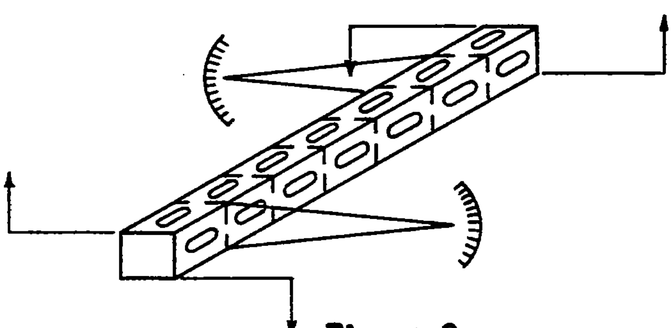

Figure 8.